A Graphical Language for Modeling Stochastic Programming

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INFORMS Annual Meeting 2003, Atlanta, GA

Problem Statement

- We wish to develop a modeling language for Stochastic Programming that:
  - Helps decision makers and analysts communicate
  - Helps the development of models from start to finish
  - Helps document existing models
  - Helps insert optimization models into information technology infrastructures
  - Is formal and complete

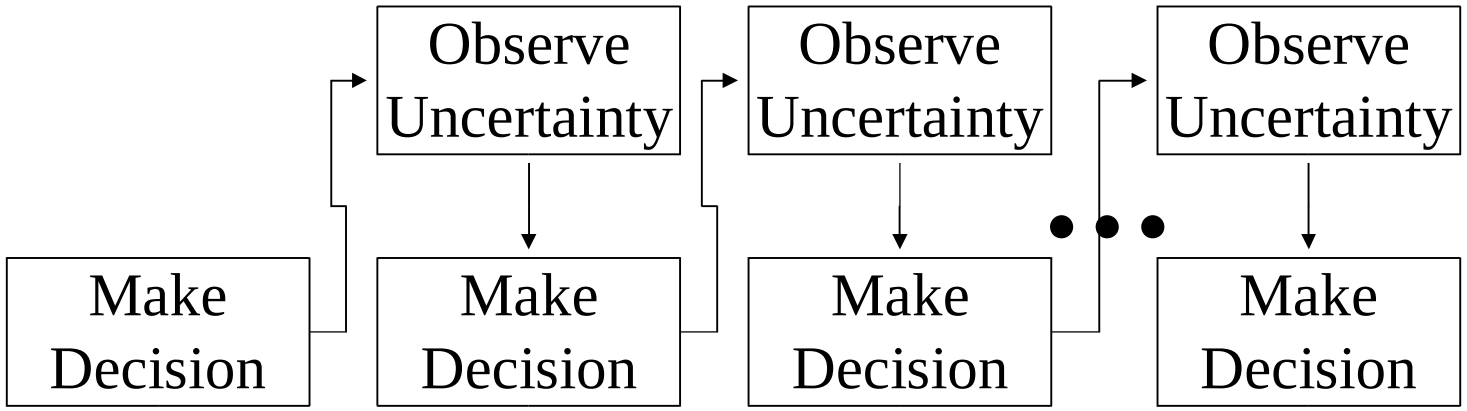
Usage Scenarios

- Elicitation
  - An OR consultant is developing a model for a business client
- Forward-Engineering
  - A team of OR experts is exploring approaches for modeling a problem
- Reverse Engineering
  - An OR expert is taking over maintenance of a model from another OR expert

Context: People

- OR Expert
  - Has expertise in developing, solving, and analyzing models
  - Has extensive technical training
- IT Expert
  - Know what information is available where
- Decision Maker
  - Knows about the problem in intimate detail

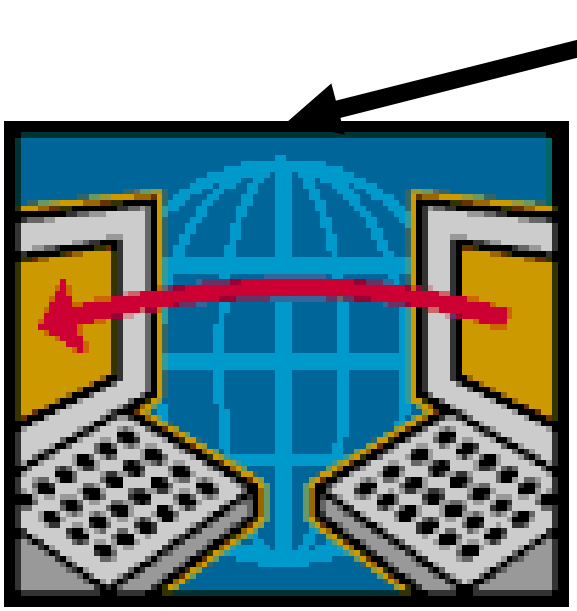
Context: Decision Framework



Context: Deliverable

$$\min c^T x + E_\xi [f (\xi , x)]$$

$$s . t \quad Ax = b$$



Summary of Tradeoffs

- Decision
  - Which graphical language to produce?
- Objective
  - Create a language that is as simple, complete, and formal as possible
- Constraints:
  - We need to support a class of sophisticated mathematical models
  - We need to provide **different views** suitable for **different audiences**

Summary of Solution

- Use the **Unified Modeling Language (UML)** as a base
  - Proven graphical language for object-oriented systems engineering
  - Has been extended to fields as varied as cryptography and Business Analysis
  - Widely used and strongly supported by the industry
  - Currently being taught to many OR, IT and Business students.

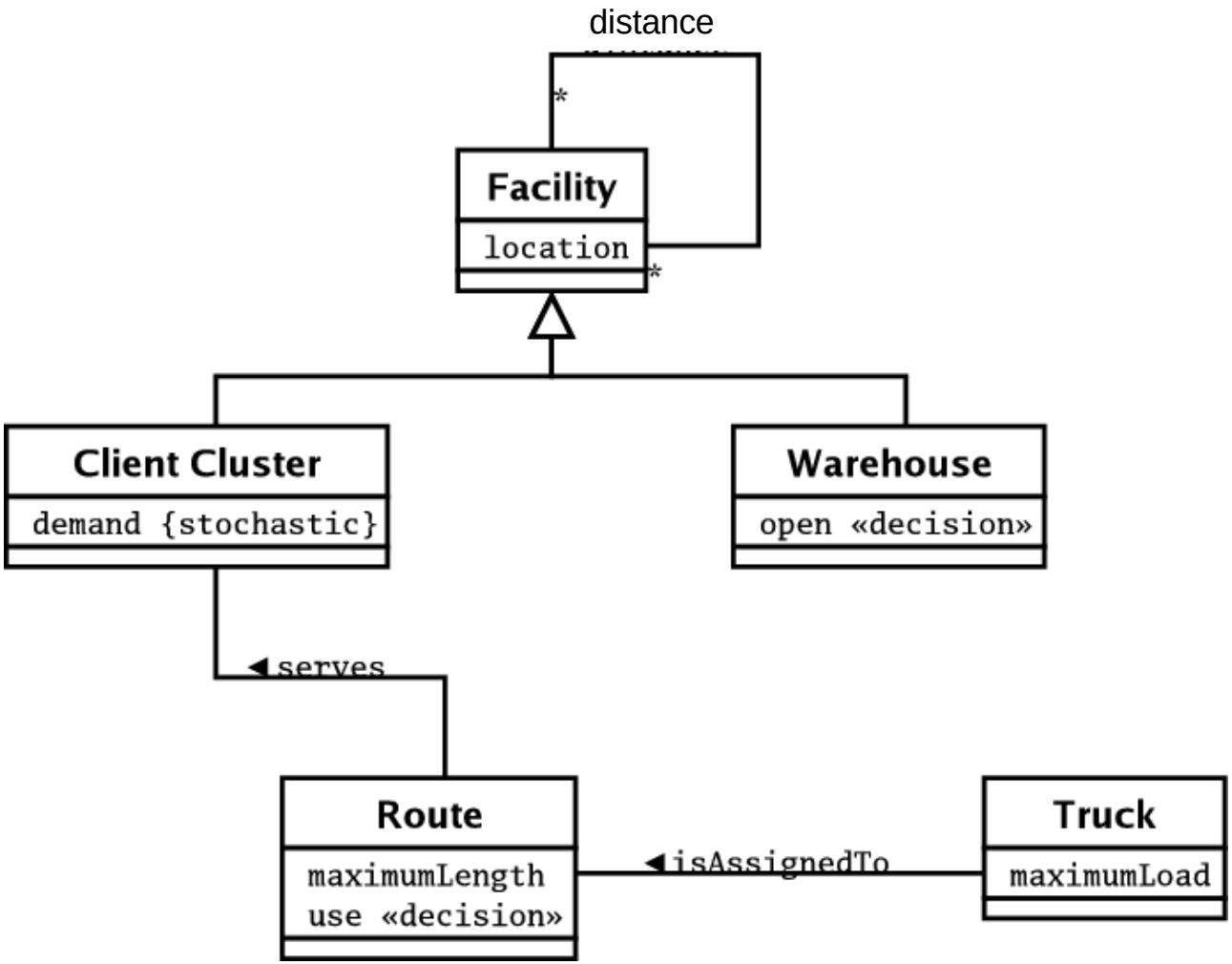
Aspects of a Stochastic Programming Problem with Recourse

- Decision Model
  - What are the decisions and constraints at each stage?
- Dynamic Model
  - What type of uncertainty is present at each stage?
  - How does the uncertainty affect the decision process?
- Interface Model
  - What are the inputs and outputs of this model?

Problem: Interface Model Issues

- Organize all the model components
  - Sets
  - Variables
  - Parameters
- Communicate the model requirements to IT professionals

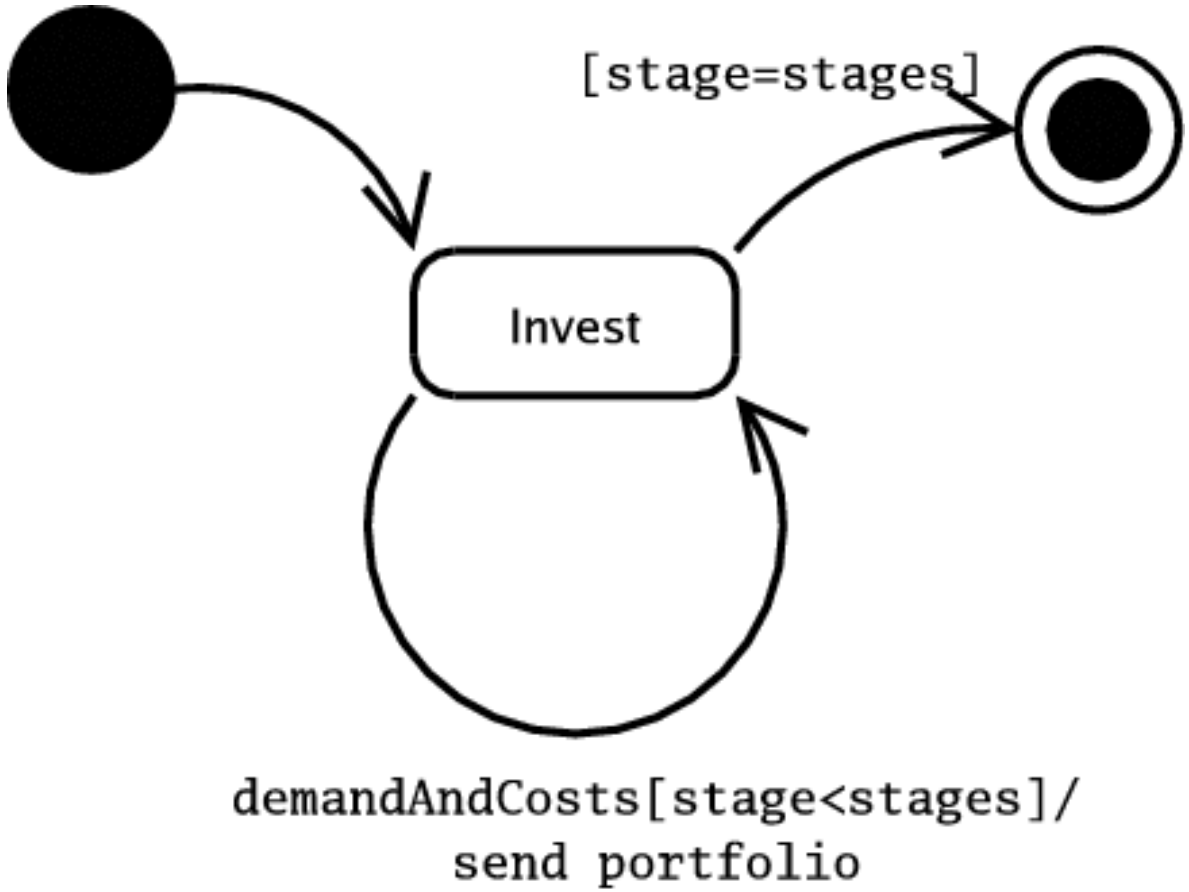
Solution: Class Diagram



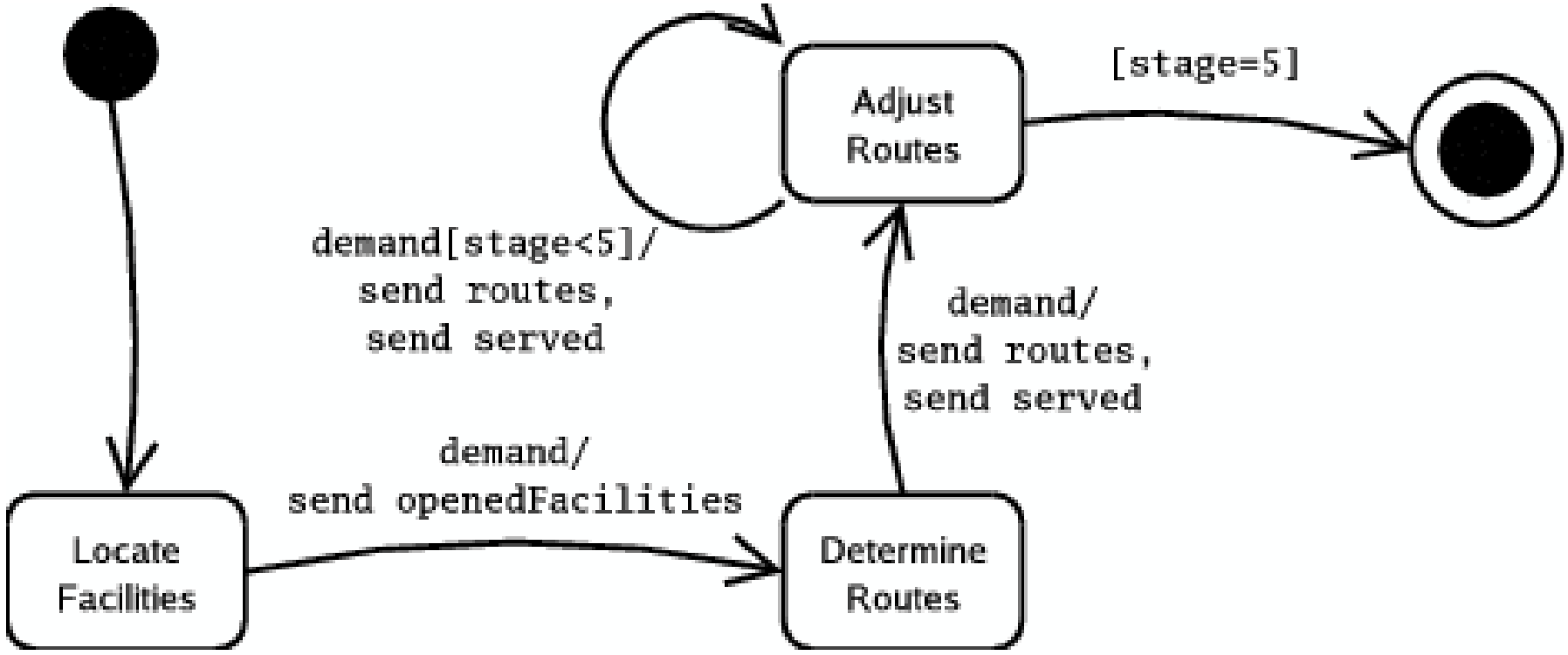
Problem: Dynamic Model Issues

- What is being modeled as uncertain?
- At what point is it observed?
- How does it affect the decision model?

Solution: Stochasticity Diagram



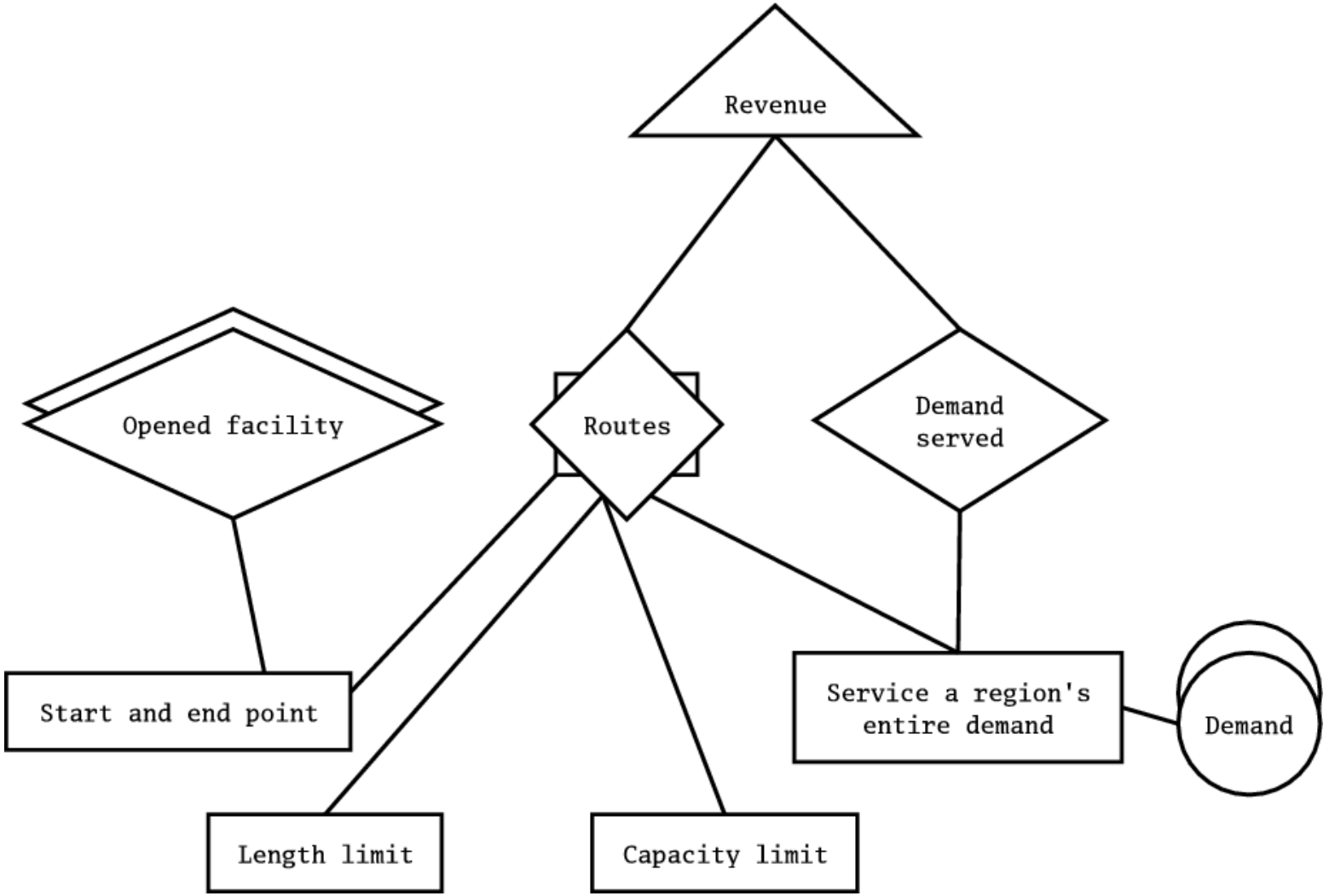
Stochasticity Diagram (continued)



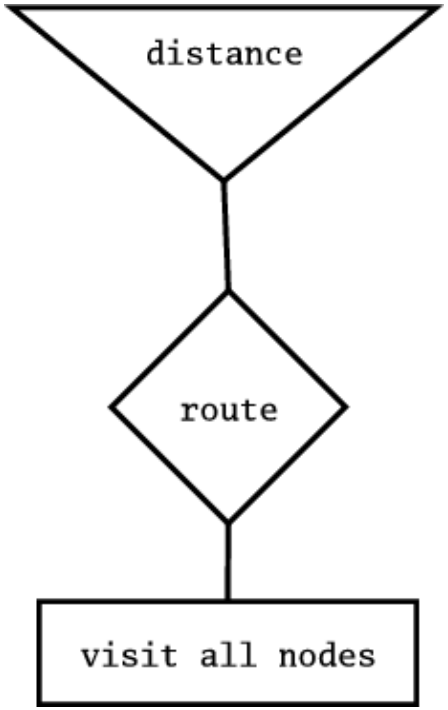
Problem: Decision Model Issues

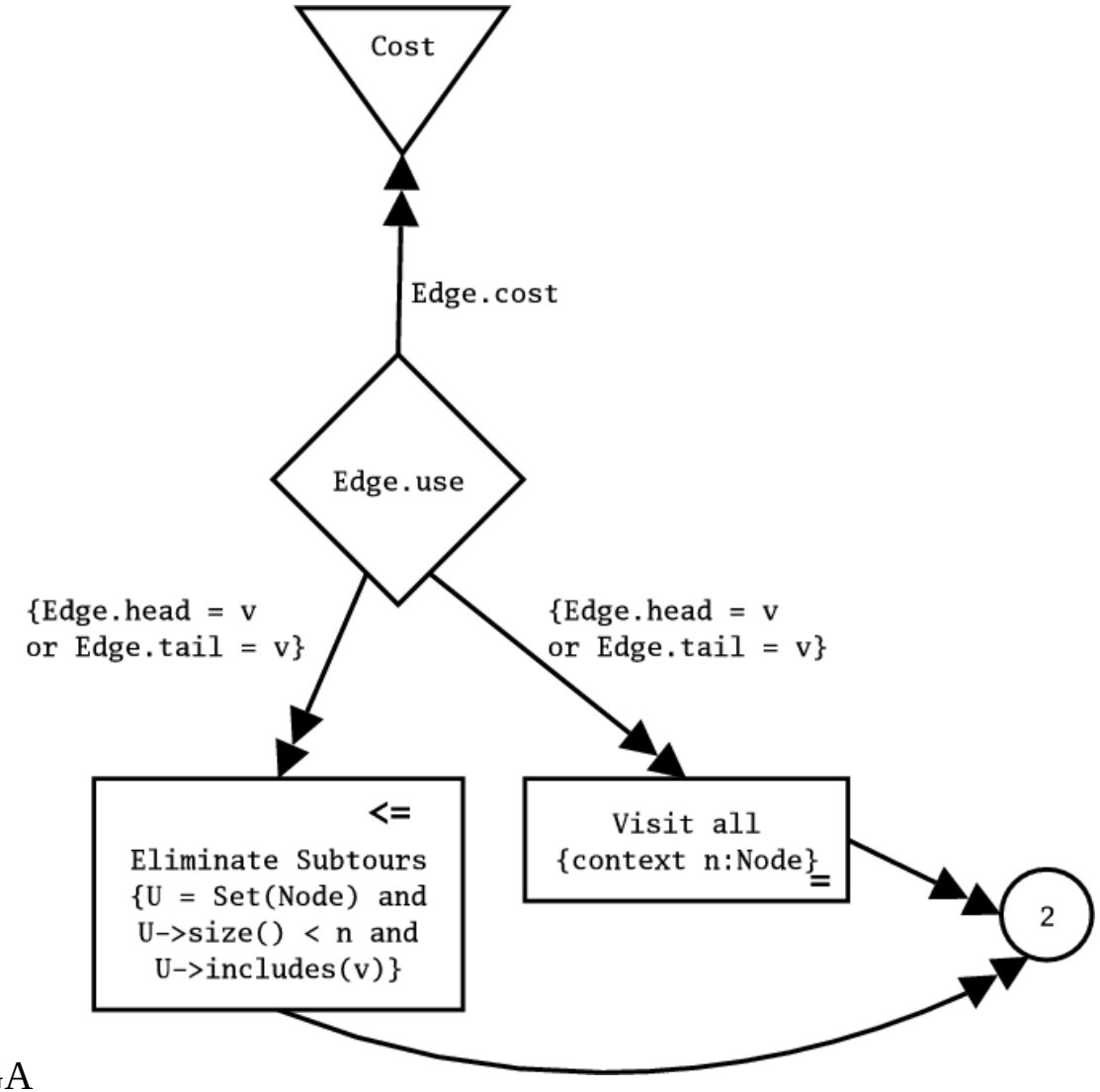
- What level of detail do we wish to represent?
- Can we (should we) produce a notation that is isomorphic to algebra?
- Simplicity x Ambiguity
- Aggregations

Solution: Element Diagram

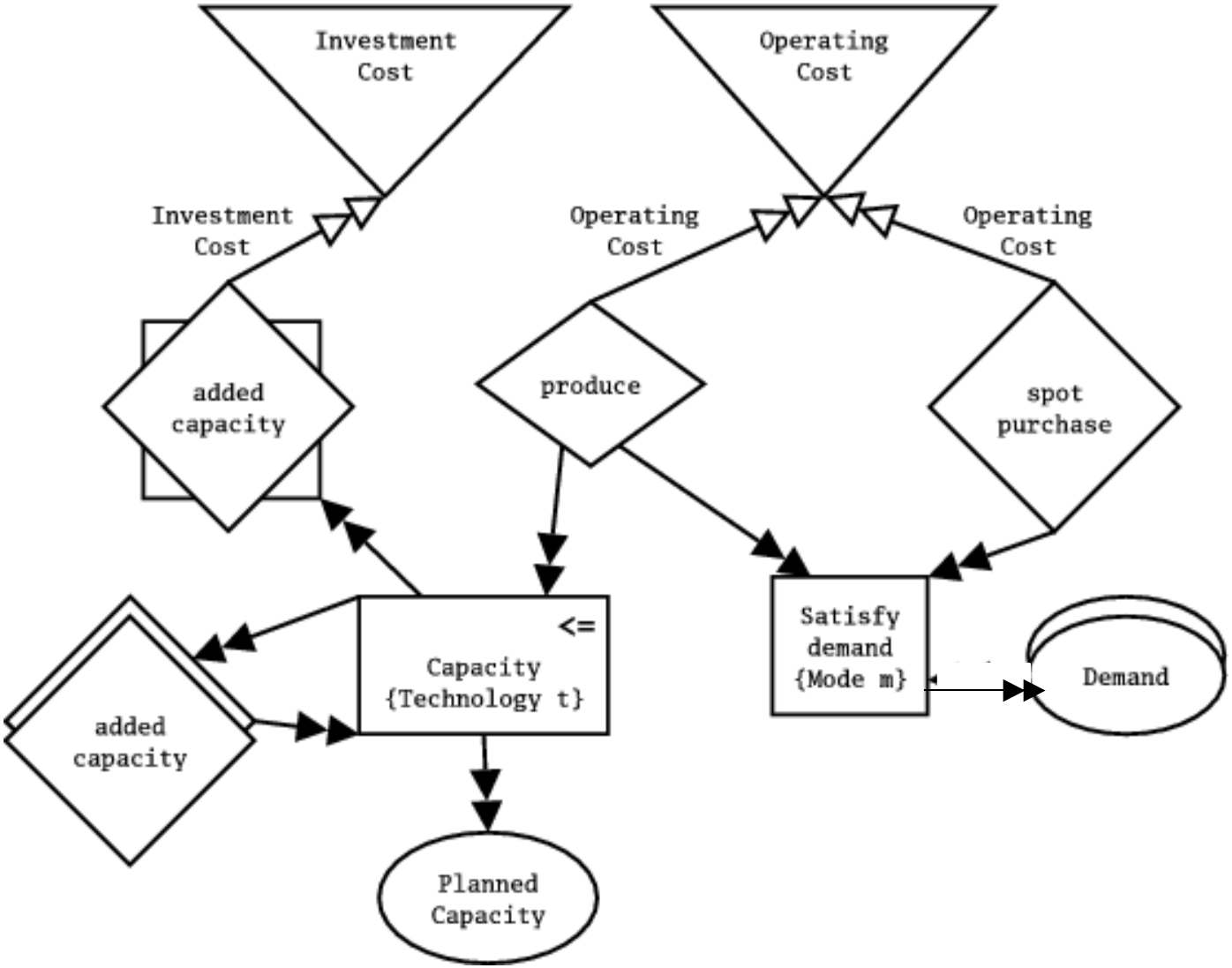


TSP Element Diagram





Element Diagram (continued)



Summary

- We have developed a language that:
  - Covers all structural aspects of Stochastic Optimization Problems
  - Is formal and formally correct
  - Is flexible, appropriate for computer implementation, and adaptable to different levels of expertise
- In the paper
  - Formal treatment of syntax, literature review and related work
  - Complete examples from the literature

Future Work

- Is it useful?
  - Human testing
  - Is implemented / supported by software vendors
  - Is taught